maintaining the temperature regime of the system with a current of 1 A; E_p, sum of the minimum energy expenditures in the realization of two separate systems; q_{op}, minimum summary heat flux at the cold end in the realization of two separate systems.

LITERATURE CITED

1. J. M. Lock, "Optimization of current leads into a cryostat," Cryogenics, December, 438-442 (1969).
2. A. G. Shifel'bain, S. P. Gorbachev, and V. K. Matyushchenkov, "Special features of sectional current leads," in: Superconductivity, Vol. 5, Atomizdat, Moscow (1977), pp. 65-68.
3. V. K. Litvinov, V. I. Kurochkin, and V. I. Karlashchuk, "Sectional optimized current leads," Inzh.-Fiz. Zh., 37, No. 2, 360-365 (1979).
4. A. S. Veselovskii, V. I. Gemintern, E. V. Lebedeva, S. S. Skvortsov, and E. S. Sklyar, "Calculation of the heat influxes in cooling two structural elements of cryostats with a common gas stream," in: Proceedings of the All-Union Electromechanical Research Institute, Moscow (1977), pp. 68-75.
5. M. P. Malkov (editor), Handbook of the Physicotechnical Foundations of Cryogenics [in Russian], Énergiya, Moscow (1973).

DETERMINATION OF THE OPTIMUM DIMENSIONS OF COOLING CHANNELS

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The article presents the formulas for calculating the optimum equivalent diameter of cooling channels in devices with high specific thermal loads.

In designing cooling systems of the most variegated machines and devices with high specific thermal loads, the most important problems are reduction of the weight and size and lowering of the energy requirements of liquid-cooling systems (LCS). An effective way of solving these problems is the optimization of the dimensions of the cooling channels.

As a rule, the liquid circuit of an LCS consists of the cooling channels of the device (machine), of communication pipes, and of channels of the heat exchanger.

The power required for pumping the heat carrier through the heat exchanger and the communication pipes is usually much lower than the power required for pumping the heat carrier through the cooling channels of the devices, and therefore the energy consumption of the LCS can be minimized chiefly by optimizing the dimensions of the cooling channels.

In the present work, the power consumption was optimized with constant mean temperature of the liquid in the cooling channels of the device and of the heat exchanger. Minimization of the power, required for pumping the heat carrier through, leads to reduced weight and size of the pump, with unchanged weight and size of the other elements of the cooling system.

Such a statement of the problem of optimizing the dimensions of the cooling channels ensures the minimization of the fundamental optimization criteria accepted at present in engineering practice the world over [1, 2]: the weight of the complex, the volume of the complex, the costs of producing it, and operating costs.

The object of the present work is to obtain dependences making it possible to determine the optimum equivalent diameter of the cooling channels and the required pump head corresponding to it ensuring with the specified values of the flow density on the surface of the cooling channel, the total surface of the cooling channels, the length of the cooling

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TABLE 1. Optimum Equivalent Diameters of Cooling Channels for Distilled Water

$\frac{q}{t_w - t_l}$, $W/m^2 \cdot deg$	l, m	ζ	$\bar{t}_l = 40°C$		$\bar{t}_l = 80°C$	
			$d_{opt} \cdot 10^3$, m	$\Delta p_{opt} \cdot 10^{-4}$, N/m^2	$d_{opt} \cdot 10^3$, m	$\Delta p_{opt} \cdot 10^{-5}$, N/m^2
$1{,}667 \cdot 10^4$	0,04	1	0,687	0,081	0,804	0,0422
		3	0,530	0,156	0,621	0,0877
		5	0,472	0,230	0,556	0,131
	0,08	1	1,202	0,107	1,401	0,557
		3	0,923	0,209	1,081	0,116
		5	0,821	0,304	0,968	0,173
	0,12	1	1,660	0,126	1,937	0,0655
		3	1,281	0,246	1,496	0,136
		5	1,140	0,358	1,339	0,203
$5 \cdot 10^4$	0,04	1	0,550	1,131	0,652	0,590
		3	0,432	2,211	0,494	1,220
		5	0,382	3,220	0,456	1,831
	0,08	1	0,862	1,482	1,121	0,778
		3	0,740	2,920	0,870	1,622
		5	0,660	4,241	0,782	2,413
	0,12	1	1,328	1,760	1,551	0,915
		3	1,024	3,441	1,202	1,903
		5	0,908	4,997	1,069	2,828
10^5	0,04	1	0,480	5,881	0,562	3,112
		3	0,372	11,72	0,436	6,468
		5	0,332	17,04	0,388	9,641
	0,08	1	0,836	7,889	0,977	4,112
		3	0,645	15,38	0,756	8,540
		5	0,574	22,38	0,676	12,68
	0,12	1	1,160	9,281	1,352	4,834
		3	0,892	18,08	1,041	10,03
		5	0,794	26,36	0,936	14,92

channels in the device, the mean temperature of the heat carrier in the channels, and the total value of the coefficient of local resistances in the LCS the specified maximum permissible temperature of the channel walls of the device with minimum energy expenditure on pumping the heat carrier through.

The length of the cooling channels in the device is usually determined by its design and functional features, and the shape and dimensions of the cross section and the number of parallel cooling channels may be varied within certain limits.

The stated problem was solved for turbulent motion of the liquid in the absence of developed surface boiling and uniform density of the heat flux over the channel perimeter.

The power expended on pumping the heat carrier through the LCS (the useful power) is determined by the expression

$$N = Q\Delta p. \tag{1}$$

With the specified total surface of the cooling channels for their given length, there is a univalent correlation between the equivalent diameter and the number of parallel channels.

For a circular channel

$$Q = 0{,}25\,\frac{dS}{l}\,w. \tag{2}$$

For turbulent motion in hydraulically smooth pipes [3]

$$\Delta p = \left(\zeta + \frac{0{,}184l}{\mathrm{Re}^{0{,}2}d}\right)\frac{\rho w^2}{2}, \tag{3}$$

where ζ is the total value of the coefficient of local resistances in the device and the LCS, reduced to the speed in the channel. For turbulent motion in the channel, with

$l/d \geqslant 50$, the mean heat-transfer coefficient is determined from the expression [3]

$$\alpha = 0{,}021 \frac{\lambda}{d} \mathrm{Re}^{0.8} \mathrm{Pr}_l^{0.43} \left(\frac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25}. \tag{4}$$

The correlation between t_w, $\bar{t}_l$, q, and α can be written in the form

$$\frac{q}{t_w - \bar{t}_l} = \frac{q}{t_w - \bar{t}_l + t_l^{\text{в}} - t_l^{\text{в}}} = \frac{1}{\dfrac{1}{\alpha} + \dfrac{t_l^{\text{в}} - \bar{t}_l}{q}}. \tag{5}$$

The expression $q/(t_w - \bar{t}_l)$ is the nominal heat-transfer coefficient reduced to the temperature difference $t_w - \bar{t}_l$. The dependence of this magnitude on α is expressed by ratio (5).

The assumption was made that the local heat-transfer coefficient at the channel outlet is equal to the mean heat-transfer coefficient. In the region of channels with the ratio $l/d > 50$ the error due to this assumption does not exceed 7% [4].

Taking (5) into account, we obtain from (4) an expression for the speed of the heat carrier in the channel in the form

$$w = \frac{\nu \mathrm{Re}^{0.2}}{0{,}021 \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25} \lambda \mathrm{Pr}_l^{0.43} \left(\dfrac{t_w - \bar{t}_l}{q} - \dfrac{l}{0{,}5\nu\rho c_p \mathrm{Re}} \right)}. \tag{6}$$

If we substitute (2) and (3) with a view to (6) into (1), we obtain after some transformations the expression for determining the useful power expended on pumping the heat carrier through, reduced to unit cooling surface:

$$\frac{N}{S} = \frac{0{,}125\rho\nu\zeta}{l} \left(\frac{\nu}{0{,}021 \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25} \lambda \mathrm{Pr}_l^{0.43}} \right)^2 \frac{\mathrm{Re}^{1.4}}{\left(\dfrac{t_w - \bar{t}_l}{q} - \dfrac{l}{0{,}5\nu\rho c_p \mathrm{Re}} \right)^2}$$

$$+ 0{,}023\rho \left(\frac{\nu}{0{,}021 \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_l} \right)^{0.25} \lambda \mathrm{Pr}_l^{0.43}} \right)^3 \times \frac{\mathrm{Re}^{0.4}}{\left(\dfrac{t_w - \bar{t}_l}{q} - \dfrac{l}{0{,}5\nu\rho c_p \mathrm{Re}} \right)^3}. \tag{7}$$

The variable parameter in expression (7) is the Reynolds number. We determine the optimum value of Re corresponding to the condition $N \to \min$. If we differentiate (7) with respect to Re, put the derivative equal to zero, and solve the expression with respect to Re, we obtain

$$\mathrm{Re}_{\mathrm{opt}} = \frac{ql}{t_w - \bar{t}_l} \left[\left[0{,}127 \left(\frac{3{,}5}{\zeta\lambda \mathrm{Pr}_l^{0.43} \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25}} - \frac{9{,}6}{\nu\rho c_p} \right)^2 - \frac{4{,}86}{\rho\nu c_p} \times \right.\right.$$

$$\left.\left. \times \left(\frac{2}{\rho\nu c_p} - \frac{8{,}75}{\zeta\lambda \mathrm{Pr}_l^{0.43} \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25}} \right) \right]^{1/2} - \frac{1{,}25}{\zeta\lambda \mathrm{Pr}_l^{0.43} \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25}} + \frac{3{,}43}{\rho\nu c_p} \right]. \tag{8}$$

In differentiating expression (7) we took the values of ζ and $\bar{t}_l$ to be constant.

The error introduced by this assumption in determining d_{opt}, with turbulent motion and the use of collectors whose cross section exceeds the total cross-sectional area of the channels, does not exceed 8%. The change of $\bar{t}_l$ in dependence on w and d is small because for efficient liquid—air heat exchangers; the heat-transfer coefficient is practically independent of the flow rate of the liquid.

If the obtained value $\mathrm{Re}_{\mathrm{opt}} < 4000$, it is necessary to change the initial data and to determine the value of d_{opt} anew.

The expressions for w_{opt}, d_{opt}, and Δp_{opt} have the form

$$w_{\mathrm{opt}} = \frac{\nu \mathrm{Re}_{\mathrm{opt}}^{0.2}}{\left(\dfrac{t_w - \bar{t}_l}{q} - \dfrac{l}{0{,}5\nu\rho c_p \mathrm{Re}_{\mathrm{opt}}} \right) 0{,}021\, \lambda \mathrm{Pr}_l^{0.43} \left(\dfrac{\mathrm{Pr}_l}{\mathrm{Pr}_w} \right)^{0.25}}, \tag{9}$$

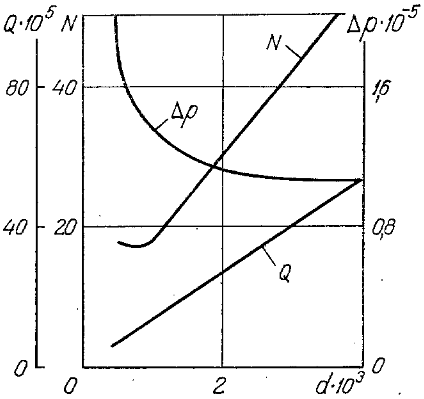

Fig. 1. Dependence of the pressure losses Δp, N/m^2, of the power expended on pumping the heat carrier through, N, W, and of the flow rate of distilled water Q, m^3/sec, on the equivalent diameter of the cooling channels d, m, for $S = 5 \cdot 10^{-3}$ m^2; $\zeta = 1$; $l = 0.08$ m; $q = 200 \cdot 10^4$ W/m^2; $t_w = 80°C$; $\bar{t}_l = 40°C$.

$$d_{opt} = \frac{\nu Re_{opt}}{w_{opt}} = Re_{opt}^{0.8}\left(\frac{t_w - \bar{t}_l}{q} - \frac{l}{0.5\nu\rho c_p Re_{opt}}\right) 0.021\,\lambda Pr_l^{0.43}\left(\frac{Pr_l}{Pr_w}\right)^{0.25}, \tag{10}$$

$$\Delta p_{opt} = \left(\zeta + \frac{0.184\,l}{Re_{opt}^{0.2}\,d}\right)\frac{\rho w_{opt}^2}{2}. \tag{11}$$

The value of d for any shape of channel is determined according to [3] by the formula

$$d = \frac{4F}{\Pi}. \tag{12}$$

With a view to (12) it is easy to prove that the values of Re_{opt}, w_{opt}, d_{opt}, and Δp_{opt} do not depend on the shape of the channel (circular, square, rectangular). With fixed length of the channels and heat-removal surface, the number of parallel cooling channels in the device changes when the shape of the channel changes, but the product $n\Pi$ remains constant. Thus the power for pumping the heat carrier through

$$N = \frac{n\Pi w d \Delta p}{4} \tag{13}$$

is the same for channels of any shape, when the equivalent diameters are equal.

The values of d_{opt} and Δp_{opt}, calculated for distilled water in a wide range of l, ζ, and $q/(t_w - \bar{t}_l)$, are presented in Table 1.

It is only natural that when the diameters of the cooling channels are selected, there are other considerations in addition to the optimization of energy consumption: technological considerations are taken into account as well as the requirements of reliability, since the diameters become smaller as a result of sediments on the channel walls which are deposited during operation of the cooling system.

All that has been explained above, and also the permissible pressure level in the devices may make it necessary to use somewhat larger cooling channels than with d_{opt}.

An investigation of the effect of deviations in the size of cooling channels toward increase, carried out in a wide range of changes of q, ζ, l, $t_w - \bar{t}_l$, showed that a deviation of d from the optimum leads to a considerable increase in power consumption for pumping the heat carrier through.

Figure 1 shows the dependences $N = f(d)$, $\Delta p = f(d)$, $Q = f(d)$ obtained for distilled water (channel with circular shape). It can be seen from $N = f(d)$ that a deviation of the equivalent diameter from the optimum leads to increased power consumption. For instance, with all other conditions being equal, an increase of d from 1 to 3 mm leads to 2.5-fold increase in the power required for pumping the heat carrier through. The pressure head decreases by a factor of 1.2 but the flow rate of heat carrier increases 2.9 times.

To determine the optimum diameter of the cooling channel, we must stipulate the values q, t_w, $\bar{t}_l$, l, and ζ. The value of ζ is specified on the basis of the construction of an analog; when devices are connected successively, it is taken that $\Sigma\zeta = z\zeta$. The hydraulic resistance of the cooling system can be taken into account by increasing the selected value of ζ by 20-30%.

When the calculations for determining d_{opt} are completed, it must be checked whether it is possible with the found d_{opt} to arrange the channels in the device that have a total surface S. If it is impossible to arrange the channels, S must be changed and d_{opt} and Δp_{opt} determined anew.

NOTATION

q, density of the heat flux, W/m^2; Δp, difference in pressures, N/m^2; S, total surface of cooling channels, m^2; l, length of cooling channels, m; $\bar{t}_l$, mean temperature of the liquid in the cooling channels, °C; t_w, maximum permissible temperature of the walls of the cooling channels, °C; N, useful power of the pump, W; Q, volumetric flow rate, m^3/sec; d, equivalent diameter of the cooling channels, m; w, mean speed of the heat carrier in the cooling channels, m/sec; Re, Reynolds number; Pr_l and Pr_w, Prandtl numbers for the heat carrier at temperatures $\bar{t}_l$ and t_w, respectively; λ, thermal conductivity, W/m·deg; ρ, density of the liquid, kg/m^3; α, mean heat-transfer coefficient, $W/m^2 \cdot deg$; $t_l{}^B$, temperature of the liquid at the outlet from the channel, °C; ν, coefficient of kinematic viscosity, m^2/sec; c_p, specific heat, J/kg·deg; F, cross-sectional area, m^2; Π, channel perimeter, m; n, number of parallel cooling channels. Subscripts: opt, optimum; l, liquid; w, wall.

LITERATURE CITED

1. V. V. Loskutov and G. S. Khordas, Calculations of the Parameters and Structural Elements of Ship Systems [in Russian], Sudostroenie, Leningrad (1968).
2. T. Conrey and D. Schneider, "Optimum selection of pumps for hydraulic networks," Trans. ASME, Ser. D. Theoretical Foundations of Engineering Calculations, No. 1, 109-116 (1976).
3. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Énergiya, Moscow (1969).
4. Handbook of Heat Engineering [in Russian], Vol. 2, Énergiya, Moscow (1976).